Using the expressions for N_{tu} and λ, equation (10) yields

$$k_{opt} = \frac{2}{1+\nu}\frac{C}{\pi D} = \frac{2C_1C_2}{\pi D(C_1+C_2)} \quad (11)$$

which may be termed as

$$\lambda_{opt} = \frac{2}{1+\nu}\frac{t}{L}. \quad (11a)$$

Equation (11) may be expressed in dimensionless form as

$$\frac{k_{opt}}{k_f} = \frac{2}{1+\nu'}\frac{Pe}{4} \quad (11b)$$

where k_f and Pe are for the fluid inside the tube and $\nu' = C_{tube}/C_{shell}$ For the case of balanced flow, the expression reduces to that of ref. [9] by substituting $\nu' = 1$.

Substituting the expressions (i)

$$\lambda_{opt} = \frac{2}{1+\nu}\frac{t}{L}$$

and, (ii)

$$\frac{1}{N_{tu,opt}} = \frac{1}{N_{tu,1}} + \frac{\nu}{N_{tu,2}} + \frac{1+\nu}{2}\frac{t}{L}$$

into equation (9), the total entropy generation rate under optimum condition may be expressed as

$$N_{S,opt} = \left[\frac{1}{N_{tu,1}} + \frac{\nu}{N_{tu,2}} + (1+\nu)\frac{t}{L}\right]I + N_{S,F}. \quad (12)$$

CONCLUSIONS

It has been shown that the optimum thermal conductivity of the partition wall in a concentric tube heat exchanger is a function of the fluid properties, flow parameters and the capacity rate ratio. It is not always advisable to provide material of high thermal conductivity. At high flow velocities copper and aluminium tubes are preferable whereas at low velocities stainless steel and even plastics provide more suitable alternatives.

Kroeger [10] has considered the effect of axial conduction in heat exchanger performance, but, has not explicitly taken into account the effect of lateral resistance. Consequently the question of an optimum thermal conductivity of the partition wall has not been investigated before.

The above analysis may also be applied with suitable modification to shell-and-tube and plate fin heat exchangers and exchangers with longitudinal fins. A shell and tube exchanger may be considered equivalent to a set of concentric tube exchangers operating in parallel.

Acknowledgements The authors are grateful to the Department of Atomic Energy, India for a fellowship to KC. This work is partially supported by the Department of Science and Technology.

REFERENCES

1. A. P. Fraas and M. N. Özişik, *Heat Exchanger Design*. Wiley, New York (1965).
2. W. M. Kays and A. L. London, *Compact Heat Exchangers*. McGraw-Hill, New York (1964).
3. R. S. Thurston, K. D. Williamson, Jr and J. C. Bronson, Cryogenic tests on a teflon tube heat exchanger. *Adv. Cryog. Engng* **13**, 574–581 (1967).
4. R. A. Gaggioli and P. J. Petit, Use the second law first, *Chemtech*. **7**, 496 (1977).
5. F. A. McClintock, The design of heat exchangers for minimum irreversibility, ASME paper No. 51-A-108 presented at the 1951 meeting of ASME (1951).
6. A. Bejan, General criterion for rating heat exchanger performance, *Int. J. Heat Mass Transfer* **21**, 655–658 (1978).
7. A. Bejan, The concept of irreversibility in heat exchanger design: counterflow heat exchangers for gas-to-gas applications, *Trans. Am. Soc. Mech. Engrs*, Series C, *J. Heat Transfer* **99**, 374–380 (1977).
8. S. Sarangi and K. Chowdhury, On the generation of entropy in a counterflow heat exchanger, *Cryogenics* **22**, 63–65 (1982).
9. K. Chowdhury and S. Sarangi, A second law analysis of the concentric tube balanced heat exchanger: Optimisation of wall conductivity, Proceedings of the 7th National Symposium on Refrigeration and Air-Conditioning, India, pp. 135–138 (1980).
10. P. G. Kroeger, Performance deterioration in high effectiveness heat exchangers due to axial heat conduction effects, *Adv. Cryog. Engng* **12**, 363–372 (1966).

Int. J. Heat Mass Transfer. Vol. 26, No. 5, pp. 786–790, 1983
Printed in Great Britain
0017-9310/83/050786-05 $03.00/0

AN ANALYSIS OF SUBSTRATE HEAT LOSSES IN STEFAN'S PROBLEM WITH A CONSTANT FLUX

BELLAVE JAYARAM
Department of Electrical Engineering
and
WILLIAM STRIEDER
Department of Chemical Engineering
University of Notre Dame, Notre Dame, IN 46556, U.S.A.

(*Received* 24 *May* 1982 *and in final form* 22 *September* 1982)

NOMENCLATURE

a, b, coefficients in the liquid layer temperature profile [K m^{-1}, K m^{-2}];
F, constant source per unit area at the substrate interface [W m^{-2}];
G_i, dimensionless flux from the substrate interface into the liquid-melting solid layer i = l or into the substrate i = s, H_i/F;
$G_{s\infty}$, asymptotic dimensionless heat flux into the substrate;
H_i, transient heat flux from the substrate interface into

the liquid–melting solid layer i = l or into the substrate i = s [W m^{-2}];

L_l, latent heat of melting per unit mass [J kg^{-1}];

t, time [s];

T, temperature [K];

T_m, initial temperature and melting temperature of the solid [K];

T_0, temperature at the substrate interface [K];

x, distance coordinate measured positive from the substrate interface [m];

x_m, position of the melting interface [m];

x_s, penetration depth of the thermal profile into the substrate [m].

Greek symbols

α_i, thermal diffusivities of the liquid i = l and substrate i = s [m^2 s^{-1}];

β, ratio of the thermal properties of the substrate and liquid, $\lambda_s\alpha_l^{1/2}/\lambda_l\alpha_s^{1/2}$;

δ, dimensionless distance coordinate, $xF/\rho_l L_l \alpha_l$;

δ_m, dimensionless thickness of the liquid layer, $x_m F/\rho_l L_l \alpha_l$;

θ, dimensionless temperature, $\lambda_l(T-T_m)/\rho_l L_l \alpha_l$;

θ_0, dimensionless substrate interface temperature, $\lambda_l(T_0-T_m)/\rho_l L_l \alpha_l$;

λ_i, thermal conductivity of the liquid i = l and the substrate i = s [W m^{-1} K^{-1}];

ρ_l, density of liquid layer [kg m^{-3}];

τ, dimensionless time, $tF^2/\rho_l^2 L_l^2 \alpha_l$.

INTRODUCTION

CONSIDER a semi-infinite layer of solid supported on a substrate layer also semi-infinite in extent, the two layers separated by a flat planar interface. Both layers are initially at the melting temperature of the upper solid layer, whose melting point, we presume, is much lower than that of the supporting substrate material. If a constant plane source of heat is suddenly applied and uniformly distributed at the substrate interface between the two layers, the upper solid will begin to melt. Thermal energy in the form of latent heat is absorbed at the moving liquid–melting solid interface. The substrate has a finite thermal conductivity, sometimes even greater than the stationary liquid layer above it, and hence throughout the process heat flux is lost into the substrate layer. This heat loss into the substrate layer varies with time, and the time average flux into the substrate is of importance in ice removal processes where thermal energy is applied at an ice–substrate interface to weaken the bonds before mechanical removal.

Problems of phase change involving melting or solidification have been studied by numerous investigators [1–9]. Since the boundary conditions at the liquid–melting solid interface are nonlinear in the temperature, exact solutions have been limited to a few simple cases. Neumann and Stefan [2] obtained the temperature distributions across the liquid layer for a fixed interface temperature. Evans *et al.* [3] have developed series expansions for small times, and Kreith and Romie [4] have performed analog solutions of melting from a constant flux at the interface with an insulating substrate. For lack of a simple direct computational technique, approximate integral methods are often employed in more complex melting problems [5].

The heat balance integral method as described by Goodman [6–8] can be applied with relative ease to problems involving phase change with accuracy sufficient for most practical situations [9]. In the present study the heat balance integral method is used to obtain melting rates and heat losses into the substrate for the melting of a semi-infinite solid on a conducting substrate, both initially at the solid melting temperature, by a constant uniform heat source at the substrate interface. Results show that in many cases the latent heat sink will significantly reduce substrate heat losses on melting.

HEAT BALANCE INTEGRALS

At the initial instant a semi-infinite layer of solid rests on a semi-infinite layer of substrate both at the melting temperature T_m of the solid. A constant external flux F of heat is suddenly applied and maintained uniformly over solid–substrate interface. The solid next to the substrate absorbs a latent heat L_l and begins to melt forming a liquid layer. The problem will involve only one spatial coordinate x measured from the origin ($x = 0$) at the substrate interface positive into the liquid–melting solid layer. Initially the liquid layer is of zero thickness and at any later time the melting interface is located at $x_m(t)$. We assume the mass density ρ_l of the liquid is a constant, equal to that of the melting solid. Heat transfer within the stacked layers is conductive, characterized by constant thermal diffusivities α_l and α_s and conductivities λ_l and λ_s, respectively, in the growing liquid and supporting substrate layers. As the initial temperature of the solid is the same as its melting temperature T_m no heat transport will occur at any time across the unmelted solid layer. While the heat source at the substrate surface is a constant, it splits into a time dependent heat flux $H_s(t)$ into the substrate and a heat flux $H_l(t)$ into the liquid–melting solid layer

$$F = H_s(t) + H_l(t). \quad (1)$$

We define for convenience the following dimensionless quantities: temperature, $\theta = (T-T_m)\lambda_l/\rho_l L_l \alpha_l$; distance, $\delta = xF/\rho_l L_l \alpha_l$; time, $\tau = tF^2/\rho_l^2 L_l^2 \alpha_l$; flux, $G_i = H_i/F$, (i = s, l); substrate parameter, $\beta = \lambda_s(\alpha_l^{1/2})/\lambda_l(\alpha_s^{1/2})$.

The substrate layer is a semi-infinite slab extending in the negative x direction from the origin at the substrate interface. The heat conduction equation for the temperature T at any time t is

$$\alpha_s \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \qquad x < 0,\ t > 0 \quad (2)$$

and the boundary condition at the substrate interface is

$$\lambda_s \frac{\partial T}{\partial x} = H_s(t) \qquad x = 0,\ t > 0. \quad (3)$$

In applying the heat balance integral method in the substrate layer, we assume a maximum penetration depth of the thermal distribution $x_s(t) < 0$ beyond which the material is in thermal equilibrium at the initial temperature T_m with no heat flux. Also we select a polynomial form for the temperature second degree in x, whose coefficients depend on time,

$$T = T_m + (T_0 - T_m)\left(\frac{x_s - x}{x_s}\right)^2 \qquad x_s \leqslant x < 0 \quad (4)$$

and $T = T_m$ for $x_s > x$, where $T_0(t)$ is the substrate interface temperature at $x = 0$. The polynomial form (4) is substituted into the conduction equation (2) and the resulting expression is integrated first over the distance from x_s to 0 and then over the time from 0 to t. The thermal penetration depth x_s is eliminated from the equation with equations (3) and (4) and the final expression [6] for the dimensionless substrate surface temperature θ_0 in terms of the dimensionless flux G_s and time τ defined above is

$$\beta\theta_0 = \left[(3/2)G_s(\tau)\int_0^\tau G_s(\tau')\,d\tau'\right]^{1/2}. \quad (5)$$

Adjacent to the substrate, conduction in the positive x-direction occurs through a liquid layer of finite thickness x_m

$$\alpha_l \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \qquad 0 < x < x_m,\ t > 0, \quad (6)$$

with the substrate interface flux condition written in terms of F and H_s from equation (1)

$$\lambda_l \frac{\partial T}{\partial x} = -H_l = H_s - F \qquad x = 0,\ t > 0. \quad (7)$$

The liquid–solid melting front at x_m advances according to

$$\rho_1 L_1 \frac{dx_m}{dt} = -\lambda_1 \frac{\partial T}{\partial x} \qquad x = x_m,\ t > 0 \quad (8)$$

where $\rho_1 L_1$ is the latent heat per unit volume. A useful alternate form of equation (8) is generated by differentiation along the melting line in x–t space and substitution of equation (6)

$$\frac{\rho_1 L_1 \alpha_1}{\lambda_1}\frac{\partial^2 T}{\partial x^2} = \left(\frac{\partial T}{\partial x}\right)^2 \qquad x = x_m,\ t > 0. \quad (9)$$

To apply the heat balance integral method [6] again we select a second degree polynomial for the temperature across the liquid layer whose coefficients a and b depend on time

$$T = T_m + a(x-x_m) + b(x-x_m)^2 \qquad 0 < x \leqslant x_m \quad (10)$$

and $T = T_m$ for $x_m < x$.

If we integrate the heat conduction equation (6) first across the liquid layer from 0 to x_m and subsequently over time from 0 to t, include the fluxes (7) and (8) from the substrate and melting solid interfaces, and reduce to dimensionless variables,

$$\tau - \int_0^\tau G_s(\tau')\,d\tau' = \int_0^{\delta_m} \theta(\delta')\,d\delta' + \delta_m. \quad (11)$$

The temperature polynomial coefficients can be related directly by equation (9), and the temperature itself written in dimensionless form,

$$\theta = \frac{a\lambda_1}{F}(\delta-\delta_m) + \frac{a^2\lambda_1^2}{2F^2}(\delta-\delta_m)^2 \qquad 0 < \delta \leqslant \delta_m. \quad (12)$$

From equation (12) the coefficient a written in terms of the dimensionless substrate surface temperature θ_0 at $\delta = 0$ is

$$a\delta_m\lambda_1/F = 1-(1+2\theta_0)^{1/2}. \quad (13)$$

The substrate boundary condition (7) provides an additional equation

$$G_s - 1 = \frac{a\lambda_1}{F} - \frac{a^2\lambda_1^2}{F^2}\delta_m. \quad (14)$$

The temperature profile (12) is substituted into equation (11), the result multiplied by $1-G_s$, then after substitution of equation (13) and finally equation (14) to eliminate a and δ_m, we have a nonlinear integral equation for the energy flux into the substrate G_s in terms of the surface temperature θ_0

$$\tau - \tau G_s(\tau) - \int_0^\tau G_s(\tau')\,d\tau' + G_s(\tau)\int_0^\tau G_s(\tau')\,d\tau'$$
$$= \tfrac{1}{3}[2+5\theta_0+2\theta_0^2-2(1+2\theta_0)^{1/2}]. \quad (15)$$

The dimensionless surface temperature θ_0, written in terms of G_s and its time integral from equation (5), form, along with equation (15), a nonlinear equation in G_s whose only parameter is the dimensionless ratio β for the liquid layer and substrate thermal properties. The depth of the melted layer δ_m can be calculated knowing G_s and its time integral from the surface temperature given by equation (5) and the combination of equations (13) and (14).

RESULTS AND DISCUSSION

For very small times G_s decreases to zero and there is no heat loss into the substrate no matter how large the conductivity. A small amount of the solid at the solid–substrate interface must melt before the interface temperature will rise above its initial solid fusion temperature, and the heat flux from the interface is initially confined entirely to the melting solid layer. On the other hand for small times, the dimensionless surface temperature θ_0 is also quite small and the RHS of equation (15) is dominated by the linear term θ_0. The product of G_s and its time integral in the LHS of equation (15) is proportional by equation (5) to the square of the surface temperature and compared to θ_0 on the RHS can be neglected. Then clearly the LHS of equation (15) is dominated by τ and from this initial equivalence of θ_0 and τ the substrate flux G_s must vary as $\beta\tau^{1/2}$. This result suggests a Taylor series expansion [3] in $\tau^{1/2}$ will provide useful estimates of G_s for $\beta\tau^{1/2} < 1$, and an expansion of the solution of equation (15) is given up to terms of order $\tau^{5/2}$

$$G_s = \beta\tau^{1/2} - \frac{40}{21}\beta^2\tau + \left(\frac{15\,665}{3528}\beta^3 - \frac{5}{4}\beta\right)\tau^{3/2}$$
$$-\left(\frac{106\,226}{9261}\beta^4 - \frac{48}{7}\beta^2\right)\tau^2$$
$$+\left(\frac{187\,092\,163}{5\,927\,040}\beta^5 - \frac{32\,621}{1120}\beta^3\right.$$
$$\left.+\frac{1589}{480}\beta\right)\tau^{5/2} - \cdots \quad (16)$$

For $\beta > 1$ the terms of the expansion alternate in sign, and for small $\beta \to 0$ the coefficients of the expansion vanish. For many physical situations with small heat flux, the expansion (16) is sufficient for the calculation of G_s, θ_0 and δ_m. In general, to obtain an estimate of the extent to which the moving latent heat sink reduces heat losses to the substrate for larger $\beta\tau^{1/2} > 1$ numerical solutions must be generated.

Equation (15) can be regarded as an implicit nonlinear first order differential equation whose dependent variable is the time integral of G_s from 0 to τ and whose first derivative is G_s. Solutions were obtained using a Newton–Raphson search for G_s, suggested for this type of equation by Collatz [10], in which the G_s integral is calculated from the standard Newton–Cotes composite quadrature formula. Accurate starter values for G_s and its time integral are necessary and were calculated directly from equation (16). The rapid initial variation of G_s over τ from 0 to 0.01 was checked against the expansion (16). In addition, long time numerical results were compared with the analytical asymptotic limit. Numerical values were also convergent to four significant figures with respect to changes in the time interval step size. The resulting curves of the instantaneous flux G_s into the substrate for various β from 0.1 to 10 and the time average heat loss $\bar{G}_s$, defined by

$$\bar{G}_s(\tau) = \frac{1}{\tau}\int_0^\tau G_s(\tau')\,d\tau', \quad (17)$$

are presented on log–log plots in Fig. 1. For the ice–liquid water system, these values of β include the range of most substrate materials of interest.

Substrate heat losses in Fig. 1, initially zero, grow to form asymptotic curves for any β. For long times, the LHS of equation (15) for the melted liquid layer is dominated by the square of the surface temperature and takes on the form given by the square of equation (5). The melted liquid layer has become semi-infinite, and the effects of the latent heat sink have vanished. The long time form of equation (15) together with equation (5) yield for the G_s asymptote of Fig. 1

$$G_{s\infty} = \frac{\beta}{1+\beta}. \quad (18)$$

This asymptote agrees with the exact solution of the heat conduction equations for semi-infinite substrate and liquid layers. For $\beta = 0.1$ the G_s curve is not yet asymptotic even at $\tau = 10$. The G_s value of 0.076 lies 16% below 0.091 that of equation (18). As a result, the time average $\bar{G}_s$ of 0.067 at $\tau = 10$, the fractional heat loss into the substrate, also lies about 12% below the instantaneous heat flux into the substrate G_s. Similar trends are found for substrates with high conductivity $\beta = 10$, at $\tau = 10$ the instantaneous heat flux 0.82 lies 10% below 0.91 of equation (18) and the average heat flux 0.78 lies 4.9% below the instantaneous value. The substrate heat loss remains

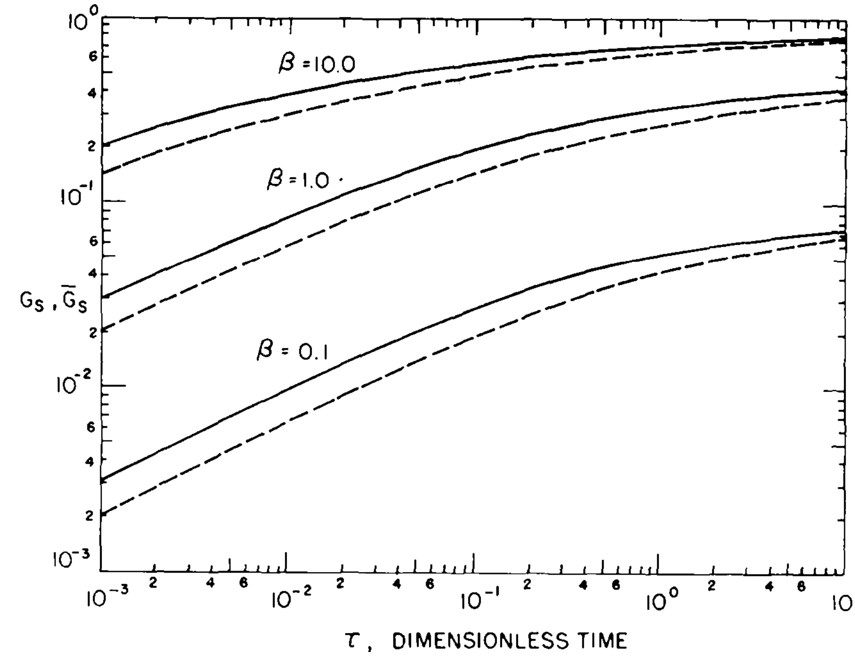

FIG. 1. Instantaneous G_s (———) and time average $\bar{G}_s$ (— — —) heat flux into the substrate at the substrate–melted solid interface. Curves are plotted vs the dimensionless time τ for various substrates, β = 0.1, 1, and 10.

significantly reduced below the long time asymptotic values even at large times, $\tau = 10$. Asymptotes are most rapidly formed at the largest β values for highly conducting substrate layers.

The surface temperature θ_0 and depth of the melted layer δ_m are obtained from equations (5), (13) and (14) and plotted versus the dimensionless time τ for the various substrate β values in Fig. 2. For the much simpler case, the limit of an insulating substrate, $H_s = 0$ and $\beta = 0$, Evans *et al.* [3] have obtained analytical solutions of equations (6)–(8) by series expansion for small τ and Kreith and Romie [4] have generated numerical solutions by analog computer. The heat balance integral results using the trial temperature [equation (10)] for $\beta = 0$ compare favorably with these exact solutions for $\tau < 10$ never differing by more than 6% in dimensionless thickness and 2% in surface temperature from the numerical solutions, and showing even better agreement with the series expansions.

These results have also been used to estimate melting times, energy requirements and substrate losses for several physical cases and approximate critical ice thicknesses for mechanical removal. For solar flux on concrete a melted thickness of 0.01

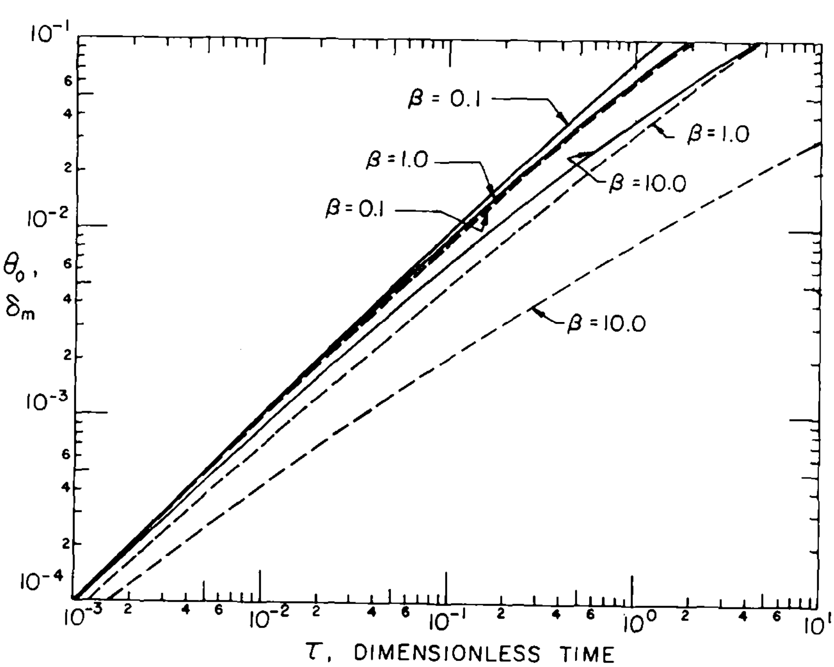

FIG. 2. Depth of the melted layer δ_m (———) and the surface temperature θ_0 (— — —) vs time τ for various substrates β = 0.1, 1, and 10.

cm is used, and we presume after reflection, absorption and scattering an approximate solar flux value F of 600 W m^{-2}. Substrate constants for concrete were taken to be $\lambda_s = 1.33$ W m^{-1} K^{-1} and $\alpha_s = 8.28 \times 10^{-7}$ m^2 s^{-1}, whereas for the water layer $\lambda_l = 0.557$ W m^{-1} K^{-1}, $\alpha_l = 0.131 \times 10^{-6}$ m^2 s^{-1} and $\rho_l L_l = 3.34 \times 10^8$ J m^{-3} for the volumetric latent heat were used. For the dimensionless critical thickness $\delta_m = 1.37 \times 10^{-3}$ it was possible to use the expansion (16) to obtain $\tau = 1.40 \times 10^{-3}$, the corresponding time of 57 s, an energy requirement of 3.4×10^4 J m^{-2} and a substrate loss of 2.3% of the incident energy. For this example $\beta = 0.95$ and substrate losses at very long times [equation (18)] can reach nearly 50%, the small substrate losses are due to the small dimensionless time value. In a second case, we assume that a radio frequency source will deliver 3.10×10^6 W m^{-2} to the outer skin of a 430 stainless steel rail. If 10^{-4} cm of ice at the rail interface must be melted for easy mechanical removal and with the substrate constants $\lambda_s = 26.2$ W m^{-1} K^{-1} and $\alpha_s = 7.32 \times 10^{-6}$ m^2 s^{-1} the dimensionless critical thickness is $\delta_m = 0.071$. From the numerical solutions of Fig. 2, the critical time $\tau = 0.13$ implies a melting time of 2.1×10^{-4} s, and an energy requirement of 6.5×10^2 J m^{-2} with a substrate energy loss of 44.4%. For this conducting rail $\beta = 6.29$ and asymptotic substrate losses (18) could reach 86% at very long times. Asymptotes form slowly enough that the latent heat sink can significantly reduce substrate heat losses during the melting process.

Acknowledgements—This work has been supported by the United States Department of Transportation, Urban Mass Transportation Administration under Contract IN-06-0009 titled Cold Weather Transit Technology Program and Report Number CWTT-21-014.

REFERENCES

1. L. Fox, What are the best numerical methods?, in *Moving Boundary Problems in Heat Flow and Diffusion* (edited by J. R. Ockendon and W. R. Hodgkins) pp. 210–241. Clarendon Press, Oxford (1975).
2. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids* (2nd edn) p. 287. Clarendon Press, Oxford (1959).
3. G. W. Evans, E. Isaacson and J. K. L. Macdonald, Stefan-like problems, *Q. Appl. Math.* **8**, 312–319 (1950).
4. F. Kreith and F. E. Romie, A study of the thermal diffusion equation with boundary conditions corresponding to solidification or melting of materials initially at the fusion temperature, *Proc. Phys. Soc.* **B68**, 277–291 (1955).
5. R. H. Tien, Freezing of semi-infinite slab with time dependent surface temperature—an extension of Neumann's solution, *Trans. AIME* **233**, 1887–1891 (1965).
6. T. R. Goodman, The heat balance integral and its application to problems involving a change of phase, *Trans. Am. Soc. Mech. Engrs* **80**, 335–342 (1958).
7. T. R. Goodman and J. J. Shea, The melting of finite slabs, *J. Appl. Mech.* **32**, 16–24 (1960).
8. T. R. Goodman, Heat balance integral—further consideration and refinements, *J. Heat Transfer* **83**, 83–86 (1961).
9. G. E. Bell, A refinement of the heat balance integral method applied to a melting problem, *Int. J. Heat Mass Transfer* **21**, 1357–1362 (1978).
10. L. Collatz, *The Numerical Treatment of Differential Equations* (3rd edn) p. 97. Springer, Berlin (1966).

Int. J. Heat Mass Transfer. Vol. 26, No. 5, pp. 790–792, 1983
Printed in Great Britain
0017-9310/83 050790-03 $03.00/0
Pergamon Press Ltd

THE INFLUENCE OF SOUND ON NATURAL CONVECTION FROM VERTICAL FLAT PLATES

HIROAKI MATSUMURA
Department of Mechanical Engineering, University of Akita, Akita 010, Japan

(*Received* 5 *September* 1982)

NOMENCLATURE

A,	width of plate;
Gr_x^*,	modified local Grashof number;
H,	height of plate;
Nu_x,	local Nusselt number without sound;
Nu_{xf},	local Nusselt number with sound;
X,	co-ordinate of height along plate measured from bottom;
f,	frequency;
q,	heat flux;
SPL,	sound pressure level [dB].

INTRODUCTION

IN 1931 ANDRADE [1] carried out experiments on the isothermal streaming of tobacco smoke in a standing wave tube by photography. In the following year, Schlichting [2] published the mathematical solution of the problem of streaming around a circular cylinder. Since then, Fand and Kaye [3], and Richardson [4] have made general surveys of the literature in this field.

As for studies of the vertical flat plate, with which the present authors are concerned, June and Baker [5] reported their experiments using a siren. In their experiments, they combined temperature difference, sound intensity and frequency, and introduced the idea of the depth of penetration of sound. They compared their results with those for heat transfer occurring without sound. However, their experiments were limited to a single flat plate. Considering the fact that the geometry of the heat transfer surface is one of the important factors on the convective field itself, it seems probable that this limitation would mean that their results could not fully explain the influence of the sound field on the convective field.

The aim of this study is to investigate experimentally the influence of a sound field on the natural convective heat transfer from vertical flat plates by means of flat plates of various width.

APPARATUS

The apparatus used for these experiments was an echo chamber made of iron plates (dimensions: thickness 1 cm; height 80 cm; width 70 cm). It was fitted with two speakers separated by 130 cm. Several plates were placed vertically with both ends fixed at the centre of the inside of the echo chamber. The plates, of Ni–Cr foil, were 0.5 mm thick, 70 cm high, and 1, 2, 4 and 10 cm wide, respectively. Experiments were made at constant heat flux. The surface temperatures were measured by 17 pairs of iron–constantan thermocouples placed at a fixed separation along the centre lines of the plates. Sound was generated by two trumpet horn speakers (100 W each) facing each other. The sound around the plates was caught by a condenser microphone. The sound frequency and pressure level were measured by a frequency analyser.